Thermally anisotropic streams: cumulative effect of the filamentation and Weibel instabilities

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Abstract

All beam plasma instabilities are dissipative, preventing the beam energy deposition. Kinetic effects arising from the perpendicular temperature of the beam or surrounding plasma could stabilize the nonresonant filamentation mode. But such beam plasma systems combine streaming and thermal anisotropic structures where the both filamentation and Weibel instabilities develop, and only clarifying the frequent confusion between these instabilities will improve the control of fusion plasma experiments. When a positive bi-Maxwellian thermal anisotropy is assumed, the two transverse instabilities, filamentation and Weibel, are emitted on the same direction and interact to yield larger growth rates. Otherwise, the destabilizing effect of the filamentation instability can be considerably reduced by supplementing with a negative thermal anisotropy. In this case, the Weibel instability arises along the streaming direction with a self-focusing contribution, and the filamentation instability is significantly delayed and suppressed.

Keywords: beam plasma instabilities, thermal anisotropy, nonresonant electromagnetic instabilities

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I. INTRODUCTION

In fusion plasma machines, collective plasma processes are responsible for particle acceleration and generation of a sufficiently strong magnetic field in regions with turbulent fields, anisotropic thermal distributions, or counterstreaming plasmas. The electromagnetic instabilities, especially those of Weibel-type, are able to generate quasistatic and large scale magnetic fields in counterstreaming or thermal anisotropic plasmas.

In his original work, Weibel (1959) has described the anisotropic temperature instability which propagates into a bi-Maxwellian plasma along the lower temperature axis ($\mathbf{k} = \mathbf{k}_{\parallel}$ and $T_{\perp} > T_{\parallel}$) (Figure 1 b). In the same year, trying to explain the physical mechanism responsible for the Weibel instability, Fried (1959) has described in fact, the counterstreaming based instability which propagates along the perpendicular direction with respect to the streams ($\mathbf{k} \perp \mathbf{v}_0$). We call it today, the filamentation instability. The mechanisms of these two instabilities have a common seed in the bi-axis anisotropy of velocity distribution function (see in Figure 1). But they are markedly different. The filamentation instability for example, can exist even within a cold counterstreaming plasma, as shown in Figure 1 a.

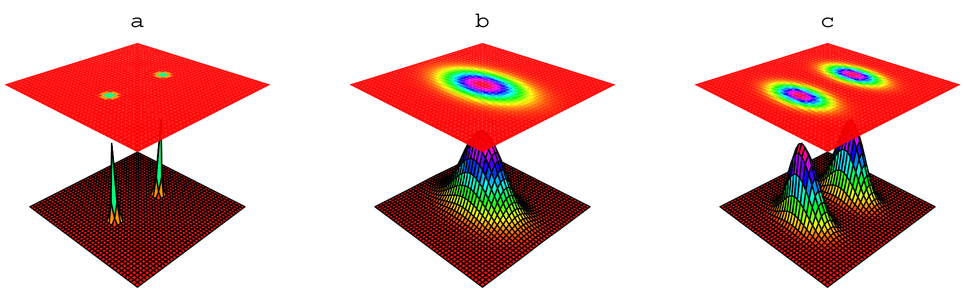

FIG. 1: Anisotropic distributions: (a) cold counterstreaming plasmas, (b) bi-Maxwellian thermal distribution, (c) each stream has a bi-Maxwellian thermal distribution.

A frequent confusion between Weibel and filamentation instabilities can be then observed, and our intention here is to clarify this question, and to provide a more realistic description of the purely growing (nonresonant) instabilities in streaming plasmas, which behave thermal anisotropies as well (see in Figures 1 c). For this purpose we consider the following distribution function [6],

$$f_0^C(v_\parallel, v_\perp) = \frac{(\epsilon_l v_{th,l} + \epsilon_r v_{th,r})^{-1}}{\pi^{3/2} v_{th,\parallel}^2} e^{-v_\parallel^2/v_{th,\parallel}^2} \left[\epsilon_l e^{-(v_\perp + v_l)^2/v_{th,l}^2} + \epsilon_r e^{-(v_\perp - v_r)^2/v_{th,r}^2}\right]. \quad (1)$$

so that both filamentation and Weibel modes can propagate perpendicularly to the streaming direction. The wave-vector direction is the preferred one in an unmagnetized plasma, and defines the parallel axis, $\parallel$, in Fig. 2, and the streams are along the perpendicular axis, $\perp$.

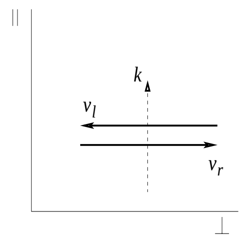

FIG. 2: Two shells of counterstreaming plasmas with velocities $v_{l,r}$. The unstable mode propagates perpendicular to the streaming direction and $\mathbf{k}$ defines the parallel direction $\parallel$.

Our choice (1) is one of the most general including different intensities $\epsilon_l \neq \epsilon_r$, and different streaming velocities $v_l \neq v_r$, for the leftward and rightward streams (the labels l and r, respectively), but it remains restricted to the charge and current neutrality condition: $\sum_a q_a n_a = 0$ and $\sum_a q_a n_a \mathbf{v}_a = 0$. A bi-Maxwellian thermal distribution is also included in (1), for each of the two counterstreams (as shown in Figure 1 c), characterized by the particle thermal velocities: $v_{th,l\parallel} = v_{th,r\parallel} = v_{th,\parallel}$, $v_{th,l\perp} = v_{th,l}$, and $v_{th,r\perp} = v_{th,r}$. Such a distribution function (1) will couple linearly the filamentation and Weibel instabilities leading to a cumulative effect of these two instabilities.

II. ELECTROMAGNETIC UNSTABLE MODES

We used the dispersion relation for the electromagnetic modes in an unmagnetized thermal plasma of arbitrary composition [4] and the distribution function (1) particularized for symmetric counterstreams, $\epsilon_l = \epsilon_r = \epsilon_0$, $v_l = v_r = v_0$, $v_{th,l} = v_{th,r} = v_{th,\perp}$ to find [3]

$$1 = N^2 + \sum_a \frac{\omega_{p,a}^2}{\omega^2} \left\{ \left[\sqrt{\pi} B_a - \frac{1}{2} e^{-B_a^2} Z'(\imath B_a)\right] \right.$$

$$\left. \times \left[\frac{1}{2}\left(B_a^2 + \frac{3}{2}\right)(A_a + 1) Z'\left(\frac{\omega}{k v_{th,\parallel,a}}\right) + 1\right] - \frac{1}{4}(A_a + 1) e^{-B_a^2} Z'\left(\frac{\omega}{k v_{th,\parallel,a}}\right) \right\}, \quad (2)$$

where the stream speed have been normalised in $B_a = v_{0,a}/v_{th,\perp,a}$, the temperature anisotropy for the component of sort a is given by $A_a = (v_{th,a\perp}^2/v_{th,a\|}^2) - 1$. $N = kc/\omega$ is the refractive index, and $\omega_{p,a} = 4\pi n_a q_a^2/m_a$ denotes the plasma frequency.

Isotropic streams: pure filamentation instability

In order to identify more clearly the pure filamentation instability, we assume the counter streams to have isotropic thermal distributions, $v_{th,\|} = v_{th,\perp} = v_{th}$, i.e. $A_a = 0$, and then the dispersion relation (2) yields

$$1 = N^2 + \sum_a \frac{\omega_{p,a}^2}{\omega^2} \left\{ \left[\sqrt{\pi} B_a - \frac{1}{2} e^{-B_a^2} Z'(\imath B_a) \right] \right.$$
$$\left. \times \left[\frac{1}{2} \left(B_a^2 + \frac{3}{2} \right) Z' \left(\frac{\omega}{k v_{th,a}} \right) + 1 \right] - \frac{1}{4} e^{-B_a^2} Z' \left(\frac{\omega}{k v_{th,a}} \right) \right\}. \quad (3)$$

Using condition $\omega \simeq \imath\omega_i = 0$ in (3) we derived the maximum value of the wave number as

$$k_M^2(B_a) = \sum_a \frac{\omega_{p,a}^2}{c^2} \left\{ \left(B_a^2 + \frac{1}{2} \right) \left[\sqrt{\pi} B_a - \frac{1}{2} e^{-B_a^2} Z'(\imath B_a) \right] - \frac{1}{2} e^{-B_a^2} \right\}. \quad (4)$$

which is strongly dependent on the streaming speed.

Bi-Maxwellian plasma: Weibel instability

For $v_l = v_r = v_0 = 0$ the counterstreaming structure disappear, $v_{th,l} = v_{th,r} = v_{th,\perp}$, $\epsilon_l = \epsilon_r$, and (1) reduces then to a bi-Maxwellian distribution function. Thus, for a bi-Maxwellian plasma, the dispersion relation (2) is reduced to

$$1 = \frac{k^2 c^2}{\omega^2} + \frac{1}{2} \sum_a \frac{\omega_{p,a}^2}{\omega^2} \left[(A_a + 1) Z' \left(\frac{\omega}{k v_{th,\|,a}} \right) + 2 \right], \quad (5)$$

which agrees exactly with equation (124) from Schlickeiser (2004) [4]. This is the classical nonrelativistic dispersion relation usually used to describe the Weibel instability in a bi-Maxwellian plasma [1]. Here the maximum value of wave-number will depend on the anisotropy as follows

$$k_M(A_a) = \left(\sum_a \frac{\omega_{p,a}^2}{c^2} A_a \right)^{1/2}. \quad (6)$$

III. CLARIFYING OF A FREQUENT CONFUSION: WEIBEL INSTABILITY IS NOT FILAMENTATION INSTABILITY!

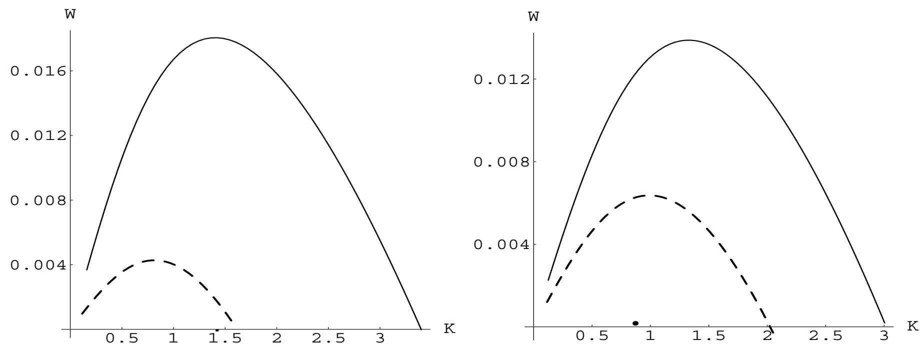

FIG. 3: Left: Cumulative effect (solid line) of Weibel (dotted line) and filamentation (dashed line) instabilities in a counterstreaming electron plasma, $B_e = v_{0,e}/v_{th,e} = 1.0$, with thermal anistropy $A_e = (v_{th,e\perp}^2 / v_{th,e\parallel}^2) - 1 = 2$. (W $= \omega_i/\omega_{p,e}$, K $= kc/\omega_{p,e}$.) Right: The same for $B_e = 1.2$, $A_e = 0.8$.

Two symmetric counterstreams are considered [3] and to each of them a thermal anisotropy is added. Such complex plasma structure is well described by the particle distribution function in (1), and dispersion equation (2), which provides the cumulative effect of both Weibel and filamentation instabilities. Numerically, the cumulative unstable mode is shown in Figure 3 by the solid line, where the maximum wave-number is

$$k_M^2(B_a, A_a) = \sum_a \frac{\omega_{p,a}^2}{c^2} \left\{ \left[\sqrt{\pi} B_a - \frac{1}{2} e^{-B_a^2} Z'(\imath B_a) \right] \left[\left(B_a^2 + \frac{3}{2} \right) (A_a + 1) - 1 \right] - \frac{1}{2} (A_a + 1) e^{-B_a^2} \right\}. \tag{7}$$

In Figure 3 (left), we have chosen first A_e and B_e so that the growing rates of the Weibel and filamentation instabilities are comparable (see the dotted and dashed lines respectively), and the cumulative effect (solid line) in this case is significantly larger than them. Otherwise, for a lower anisotropy $A_e = 0.8$, Figure 3 (right), the cumulative growth rate (solid line) remains two times larger than that of filamentation instability (dashed line).

With a negative anisotropy $A_e < 0$ (i.e. $T_{e,\perp} < T_{e,\parallel}$), no Weibel instability occurs along the parallel axis in Figure 2. In this case the effective anisotropy is reduced, and therefore the cumulative effect tends to suppress the unstable mode shown in Fig. 4 by the solid line.

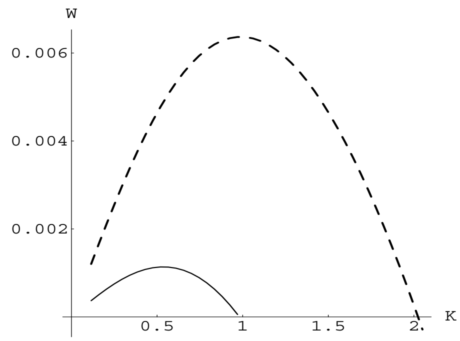

FIG. 4: The same as in Fig. 3 with $B_e = 1.2$, $A_e = -0.5$. No Weibel effect exist along the direction of filamentation instability (dashed line), which is suppressed by the cumulative effect (solid line).

IV. CONCLUSION

Quasistatic magnetic fields are originary induced by one or both of the electromagnetic instabilities: either due to a thermal anisotropy when the Weibel instability develops [1], or in a beam or counterstreaming plasma structure where the filamentation instability arises [2]. For a positive thermal anisotropy the filamentation and Weibel instabilities, are growing on the same direction and interact to yield larger growth rates. Otherwise, the destabilizing effect of the filamentation instability can be considerably reduced for a negative anisotropy, when Weibel instability arises along the streaming direction with a self-focusing contribution, and the filamentation instability is significantly delayed and even suppressed.

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