Moment approach to the derivation of general parallel closures

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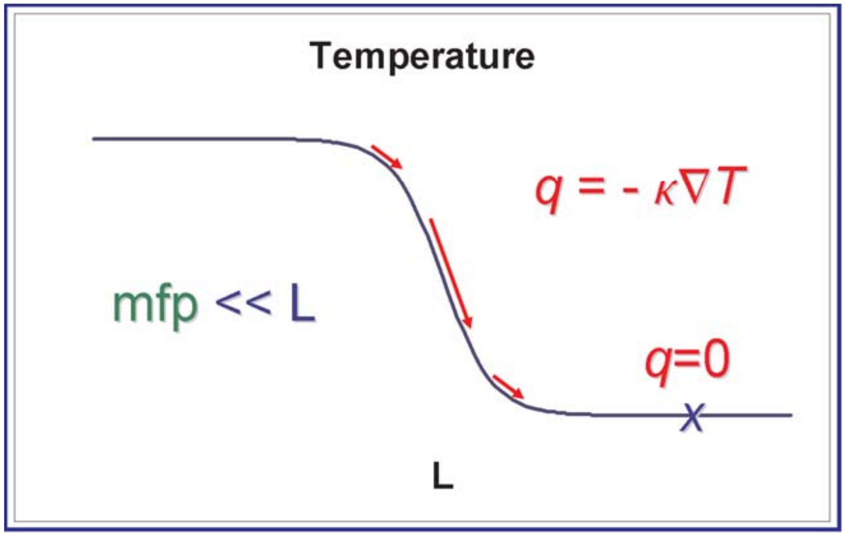
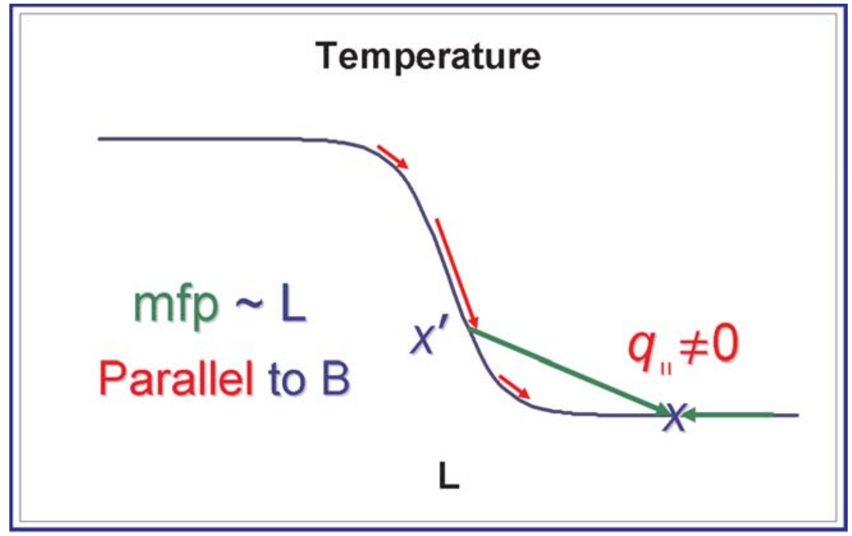
Outline

- Diffusive closure vs. integral closure
- SSPX simulations using diffusive closure vs. integral closure
- General moment equations with exact linearized Coulomb collision operator
- Derivation of general parallel closures
- Convergence (truncation independence) of the closures
- Example calculations of the general heat flux closure
- NIMROD implementation

Braginskii closure for heat flux: why integral closure?





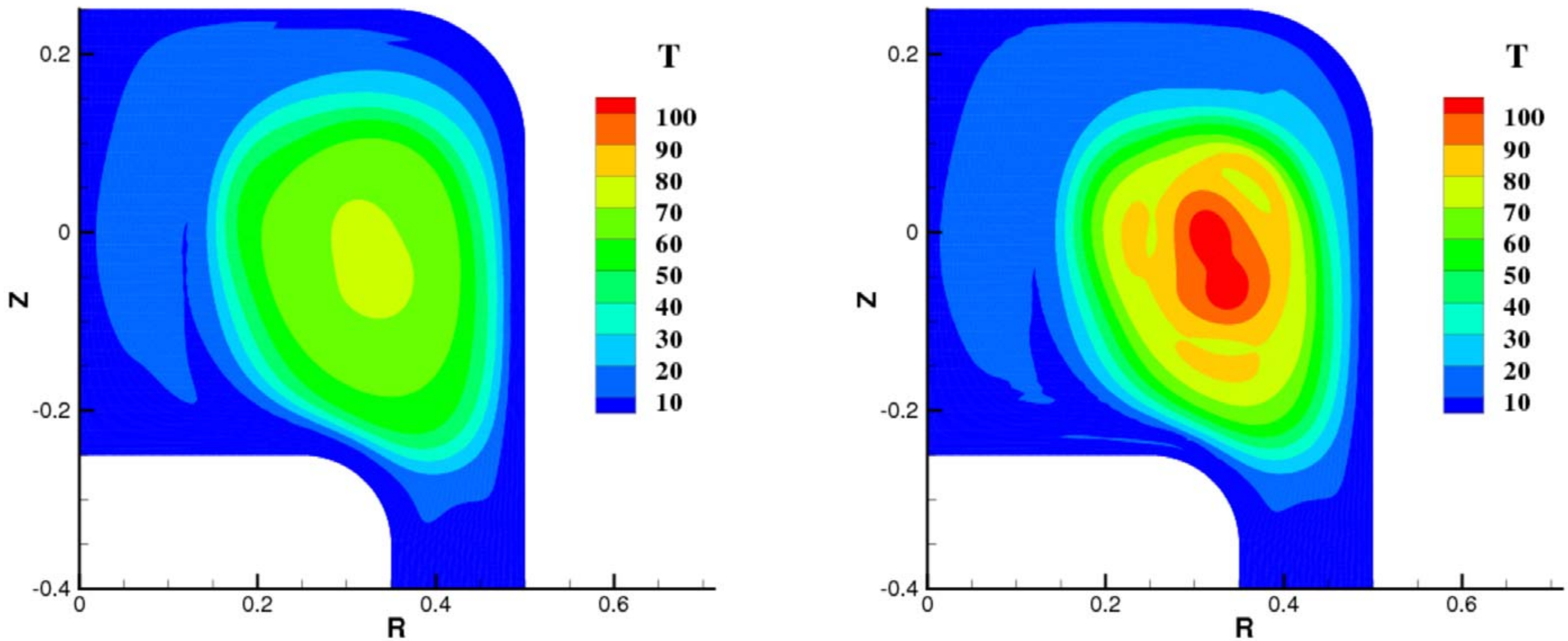
- Integral heat flux (Held *et al.*, PoP 2001):
  - Derived from the pitch-angle scattering operator
  - Kernel involves the velocity integral

$$\mathbf{q}_{\parallel} = \int_{L}^{\pm\infty} dL' \nabla_{\parallel} T(L') K_{\pm}(L, L')$$

$$K_{\pm}(L, L') = \frac{1}{v_T^3} \int d^3 v \left(L_1^{(3/2)}\right)^2 e^{-v^2/v_T^2} P_1^2(\xi) \sum_{i=1}^{N} a_i e^{\pm|\bar{k}_i|(L-L')}$$

Temperature simulations in SSPX

- Braginskii closure vs. integral closure: The integral closure yields more consistency with experimental observations



Improvement of integral closures with full linearized collision operator

- Fast short-scale gyrating motions averaged out (strong magnetic field)
- Stationary drift kinetic equation for parallel closures

$$\begin{pmatrix} 0 & \Psi_1 & & & \\ \Psi_1^T & 0 & \Psi_2 & & \\ & \Psi_2^T & 0 & \Psi_3 & \\ & & \Psi_3^T & 0 & \\ & & & & \ddots \end{pmatrix} \begin{pmatrix} \partial_L M^0 \\ \partial_L M^1 \\ \partial_L M^2 \\ \partial_L M^3 \\ \vdots \end{pmatrix} = \begin{pmatrix} C^0 M^0 \\ C^1 M^1 \\ C^2 M^2 \\ C^3 M^3 \\ \vdots \end{pmatrix} + \begin{pmatrix} \partial_L V_\parallel \\ \partial_L T \\ 0 \\ 0 \\ \vdots \end{pmatrix}$$

- Exact linearized Coulomb collision integrals (Ji and Held, PoP 2006) are used

$$\int d\mathbf{v} \mathrm{P}^l(\mathbf{s}_a) L_k^{(n+1/2)}(s_a^2) C(f_a, f_b) = \sum_k (A_{ab}^{lpk} \mathrm{M}_a^{lk} + B_{ab}^{lpk} \mathrm{M}_b^{lk})$$

Derivation of general parallel closures

- Truncated $N \times N$ matrix equation: $\Psi \partial_L \mathrm{M} = C\mathrm{M} + \mathrm{G}$
- Diagonalize $\partial_L \mathrm{M} = \Psi^{-1} C\mathrm{M} + \Psi^{-1}\mathrm{G}$

$$\partial_L \mathrm{m} = W^{-1}\Psi^{-1} CW\mathrm{m} + \mathbf{g}$$

$$\mathrm{m} = W^{-1}\mathrm{M}, \ \mathbf{g} = W^{-1}\Psi^{-1}\mathrm{G}$$

- Decoupled ordinary differential equations

$$\partial_L m_i = k_i m_i + g_i$$

$$m_i^{\pm}(L) = \int_{\pm\infty}^{L} g_i(L') e^{\pm|k_i|(L-L')} dL'$$

- Closures

$$\mathrm{M} = W\mathrm{m}$$

$$q_{\|} = M^{11} = \sum_j W_{qj} m_j$$

Convergence of the general closures

- General form of the heat flux closure

$$q_{\|}(L) = \int_0^\infty K(L, L') \frac{dT(L')}{dL'} dL'$$

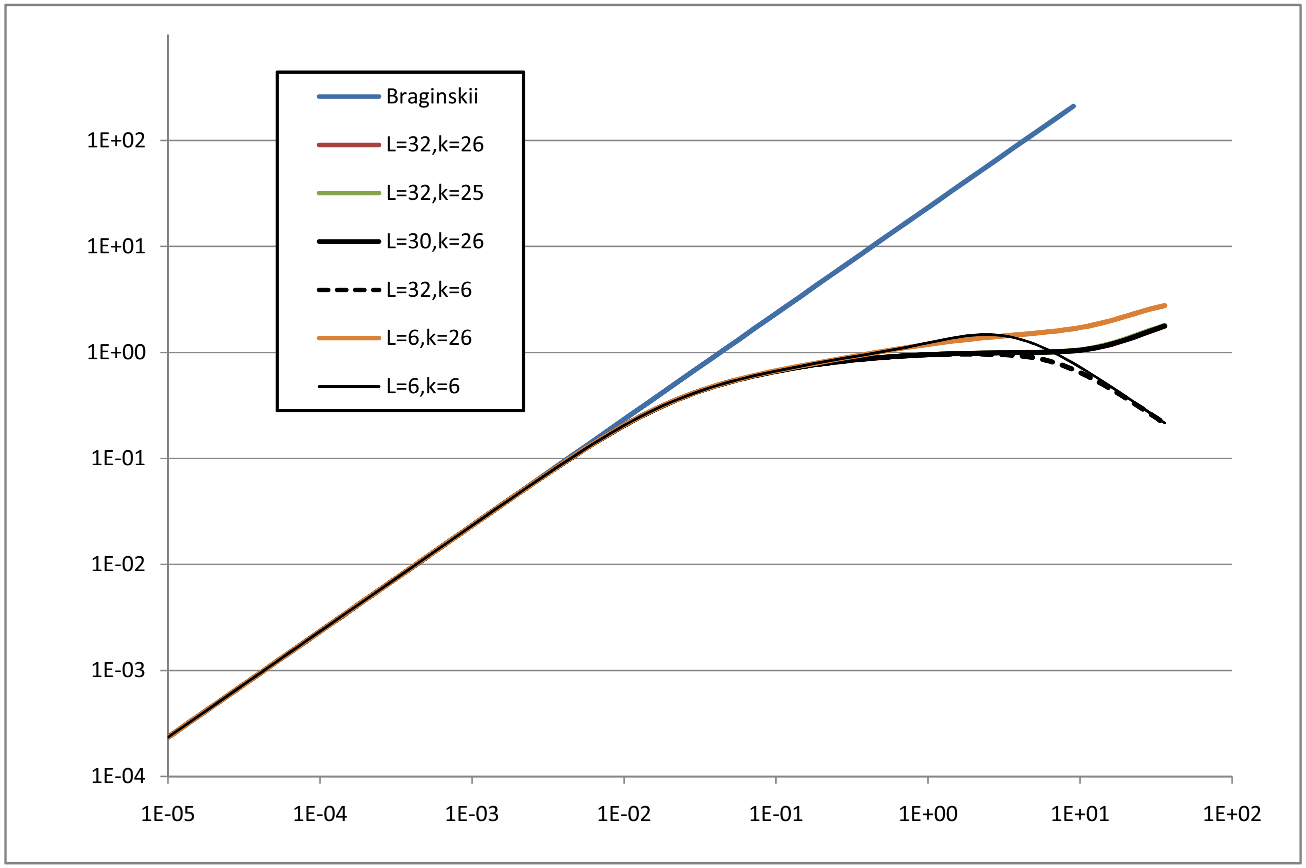
where the kernel function is

$$K(L, L') = \sum_i c_i e^{-|k_i|(L'-L)}.$$

- The physical results should be truncation-independent.
- For a given collisionality L_{mfp}/L_T, there exist l and k such that the closure calculations do not change by $l+2$ and $k+1$ calculation.
- The longer mean free path requires the larger l and k.
- Necessary number of moments in the truncation closure scheme.

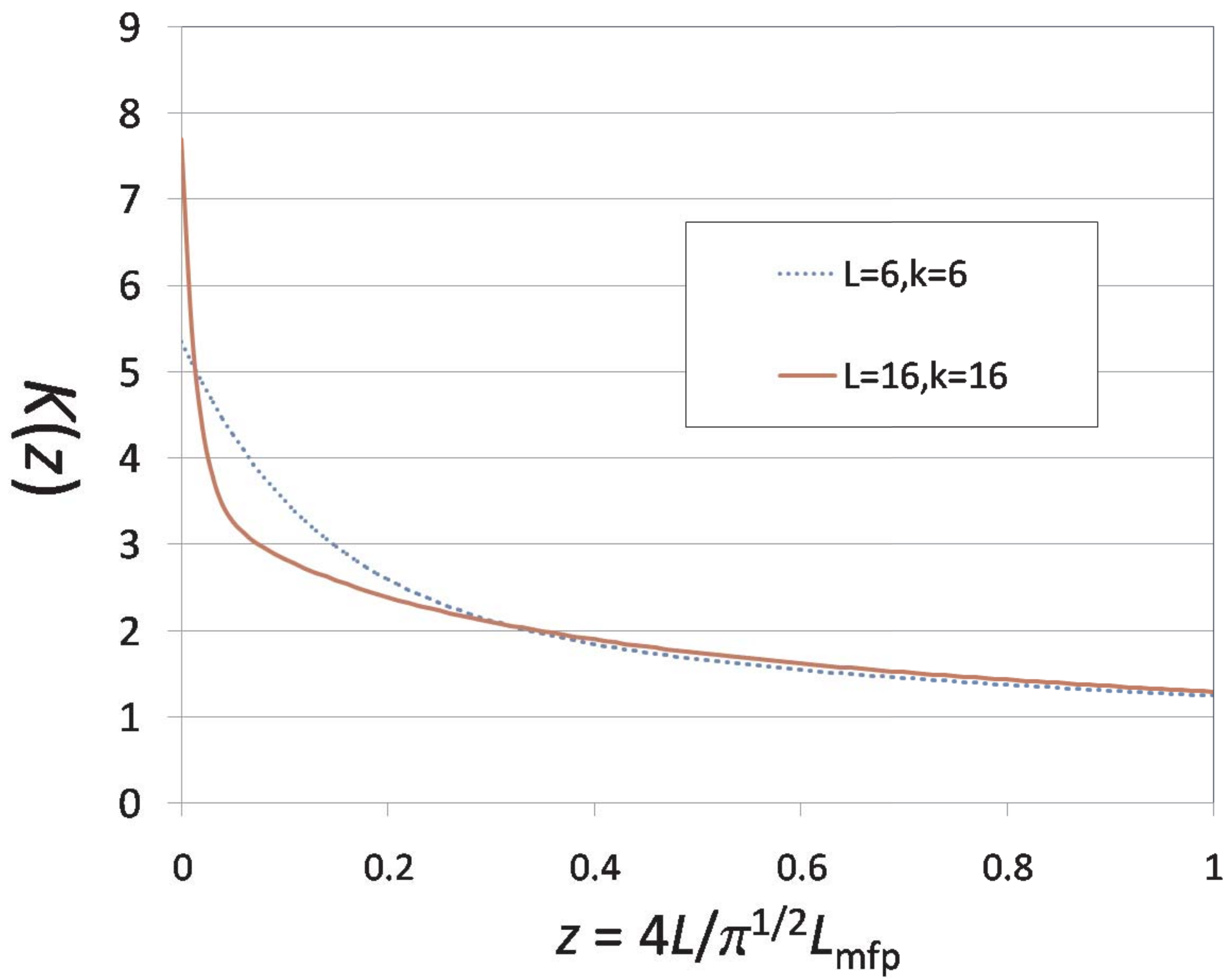
Lmfp/LT	Braginskii	L=32,k=26	L=32,k=25	L=30,k=26	L=32,k=6	L=6,k=26	L=6,k=6	Braginskii/L32k32 (%)
1.00E-05	0.000233	0.000233	0.000233	0.000233	0.000233	0.000233	0.000233	0.0
4.53E-05	0.001056	0.001055	0.001055	0.001055	0.001055	0.001055	0.001055	0.1
5.67E-05	0.001324	0.001328	0.001328	0.001328	0.001328	0.001328	0.001328	-0.3
7.18E-05	0.001676	0.001672	0.001672	0.001672	0.001672	0.001672	0.001672	0.2
9.02E-05	0.002103	0.002105	0.002105	0.002105	0.002105	0.002105	0.002105	-0.1
0.000113	0.002648	0.00265	0.00265	0.00265	0.00265	0.00265	0.00265	-0.1
0.000143	0.003335	0.003335	0.003335	0.003335	0.003336	0.003336	0.003336	0.0
0.00018	0.004198	0.004199	0.004199	0.004199	0.0042	0.0042	0.0042	0.0
0.000227	0.005287	0.005286	0.005286	0.005286	0.005287	0.005287	0.005287	0.0
0.000285	0.006653	0.006655	0.006655	0.006655	0.006655	0.006655	0.006655	0.0
0.000359	0.008379	0.008377	0.008377	0.008377	0.008378	0.008378	0.008378	0.0
0.000452	0.010549	0.010544	0.010544	0.010545	0.010545	0.010545	0.010545	0.0
0.000569	0.013281	0.013272	0.013271	0.013272	0.013273	0.013273	0.013273	0.1
0.000717	0.016717	0.016702	0.016702	0.016703	0.016704	0.016704	0.016704	0.1
0.000902	0.021049	0.021015	0.021015	0.021016	0.021018	0.021018	0.021018	0.2
0.001136	0.026495	0.026433	0.026433	0.026434	0.026436	0.026437	0.026437	0.2
0.00143	0.033358	0.033233	0.033233	0.033233	0.033236	0.033236	0.033236	0.4
0.0018	0.041997	0.041748	0.041748	0.04175	0.041753	0.041753	0.041753	0.6
0.002266	0.052873	0.052384	0.052383	0.052385	0.052389	0.052389	0.052389	0.9
0.002853	0.066556	0.065609	0.065609	0.065611	0.065616	0.065616	0.065616	1.4
0.003592	0.083792	0.081953	0.081952	0.081955	0.081961	0.081961	0.081961	2.2
0.004522	0.105486	0.101966	0.101965	0.101968	0.101975	0.101976	0.101976	3.5
0.005693	0.132802	0.12617	0.126168	0.126172	0.126181	0.126181	0.126181	5.3
0.007167	0.16719	0.154967	0.154964	0.154969	0.15498	0.154979	0.15498	7.9
0.009022	0.210478	0.188536	0.188531	0.188538	0.188553	0.188549	0.188553	11.6
0.011358	0.264976	0.226743	0.226736	0.226745	0.226764	0.226756	0.226764	16.9
0.014299	0.333585	0.269092	0.26908	0.269093	0.269104	0.269105	0.269106	24.0
0.018001	0.419958	0.314752	0.314737	0.314752	0.314729	0.314767	0.314736	33.4
0.022662	0.528695	0.36267	0.362652	0.36267	0.362617	0.362696	0.362636	45.8
0.02853	0.665586	0.411726	0.411706	0.411725	0.411735	0.411786	0.41179	61.7
0.035917	0.837922	0.460893	0.460873	0.460892	0.461064	0.461045	0.461225	81.8
0.045218	1.054885	0.509358	0.509341	0.509358	0.509581	0.509737	0.510022	107.1
0.056925	1.328013	0.556577	0.556564	0.556578	0.556495	0.557455	0.557518	

0.071665	1.671871	0.602241	0.602233	0.602244	0.601625	0.604143	0.603693		
0.090221	2.104767	0.646201	0.646196	0.646204	0.64538	0.650051	0.649291		
0.113581	2.649743	0.688353	0.68835	0.688355	0.688084	0.695662	0.695401		
0.14299	3.335824	0.728547	0.728543	0.728546	0.729233	0.741603	0.74272	L32k25/L32k26 (%)	L30k26/L32k26 (%)
0.180013	4.199554	0.766531	0.766526	0.766528	0.767524	0.788547	0.791135	0.00	0.00
0.226624	5.286925	0.801967	0.801961	0.801961	0.801804	0.83712	0.840052	0.00	0.00
0.285302	6.655845	0.834486	0.834478	0.834477	0.832118	0.887798	0.889277	0.00	0.00
0.359174	8.379215	0.863768	0.86376	0.863758	0.859772	0.940794	0.93975	0.00	0.00
0.452173	10.5488	0.889619	0.889613	0.88961	0.886196	0.99596	0.993572	0.00	0.00
0.569253	13.28016	0.912011	0.912006	0.912002	0.911565	1.052733	1.053394	0.00	0.00
0.716647	16.71873	0.931067	0.93106	0.931059	0.934309	1.110126	1.121563	0.00	0.00
0.902205	21.04763	0.947038	0.947035	0.947032	0.951957	1.166884	1.199086	0.00	0.00
1.135809	26.49741	0.960268	0.960271	0.960264	0.962773	1.221743	1.28398	0.00	0.00
1.429899	33.35826	0.971118	0.971117	0.971118	0.967077	1.273683	1.368985	0.00	0.00
1.800135	41.99555	0.979932	0.979928	0.97994	0.967045	1.322193	1.440008	0.00	0.00
2.266236	52.86927	0.987088	0.987106	0.987121	0.964662	1.36755	1.477808	0.00	0.00
2.853022	66.55847	0.992959	0.992999	0.993056	0.958983	1.410782	1.464594	0.00	0.01
3.591742	83.79215	0.997759	0.997765	0.997995	0.944728	1.453255	1.392998	0.00	0.02
4.521736	105.4881	1.001717	1.001756	1.002223	0.913925	1.496571	1.270683	0.00	0.05
5.692528	132.8016	1.006133	1.006788	1.007109	0.860355	1.54345	1.116475	0.07	0.10
7.166468	167.1873	1.014898	1.017543	1.016592	0.783865	1.599159	0.95162	0.26	0.17
9.022049	210.4764	1.035081	1.041721	1.03775	0.690993	1.672265	0.792819	0.64	0.26
11.35809	264.9741	1.075799	1.088517	1.079665	0.591663	1.773644	0.649923		
14.29898	333.5826	1.145966	1.166275	1.151206	0.495024	1.913405	0.526943		
18.00135	419.9556	1.25201	1.280229	1.258743	0.407158	2.096189	0.424123		
22.66236	528.6927	1.395749	1.430314	1.40401	0.330911	2.31579	0.339744		
28.53022	665.5847	1.571975	1.608593	1.581648	0.266782	2.55042	0.271318		
35.91742	837.9215	1.765588	1.796523	1.776308	0.21394	2.761293	0.21625		
Lmfp/LT	Braginskii	L=32,k=26	L=32,k=25	L=30,k=26	L=32,k=6	L=6,k=26	L=6,k=6	L32k25/L32k26 (%)	L30k26/L32k26 (%)



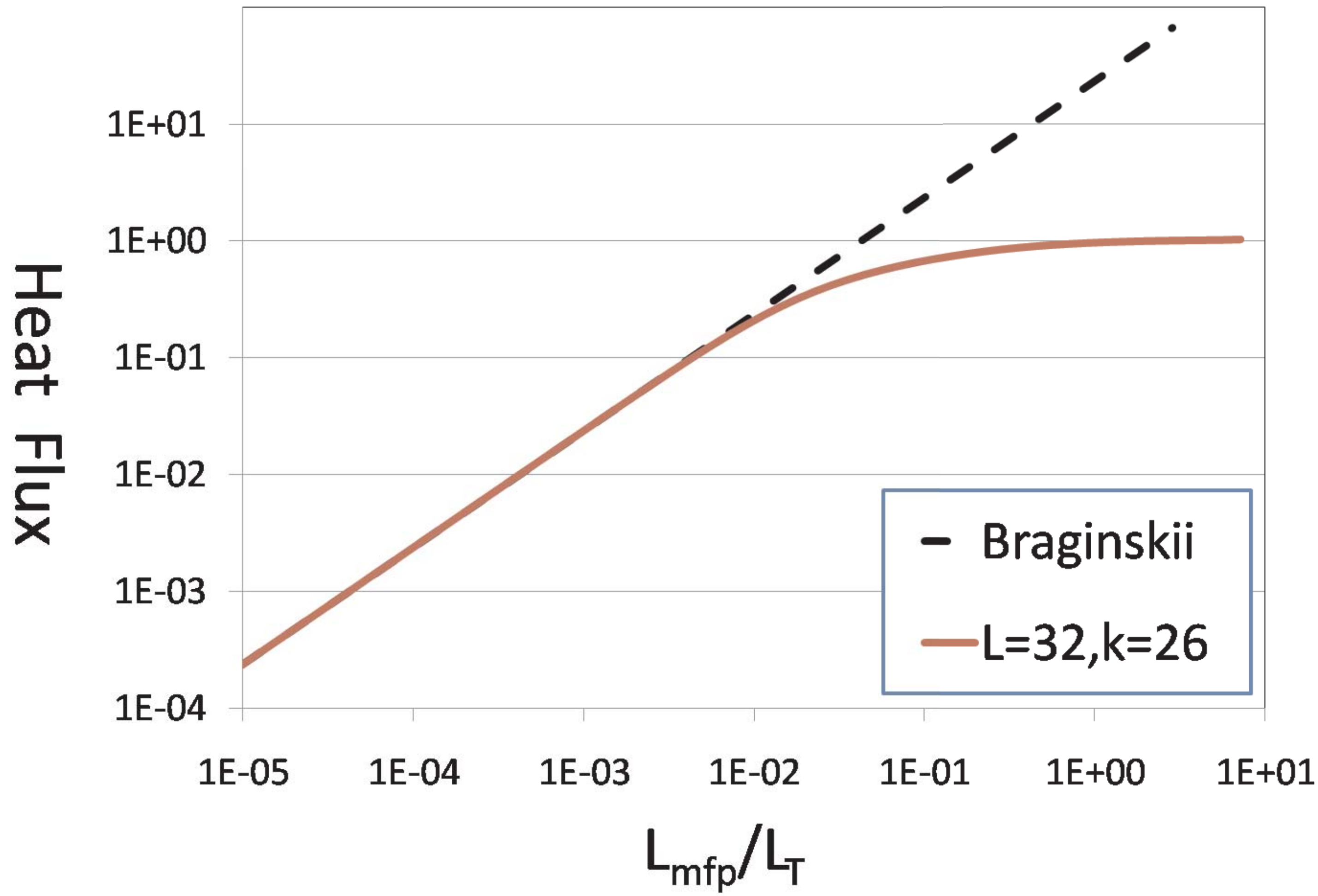
Braginskii
L=32,k=26
L=32,k=25
L=30,k=26
L=32,k=6
L=6,k=26
L=6,k=6
1E+02
1E+01
1E+00
1E-01
1E-02
1E-03
1E-04
1E-05
1E-04
1E-03
1E-02
1E-01
1E+00
1E+01
1E+02

Kernel functions for the general heat flux closure



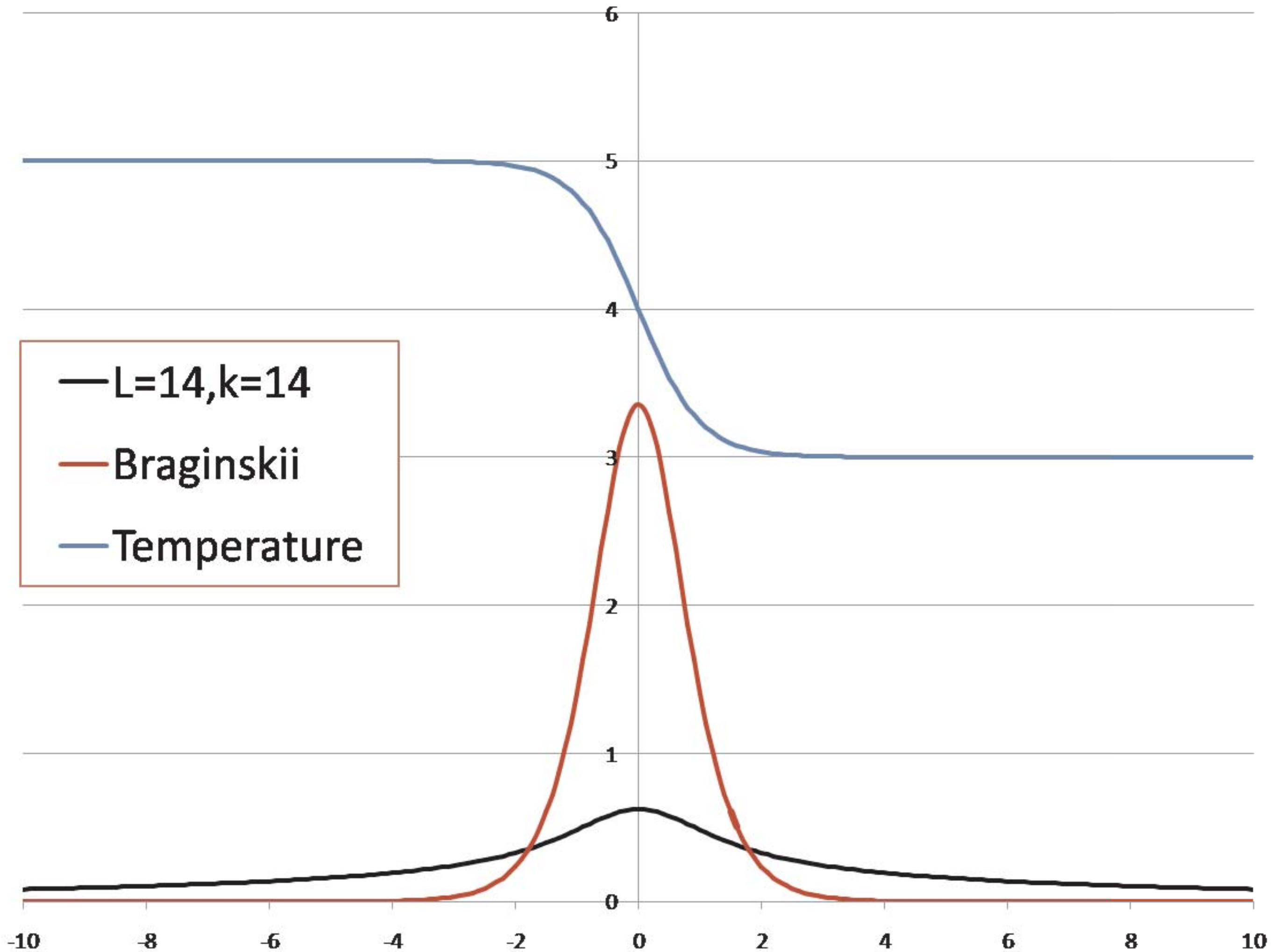
- $\int_0^\infty K(z)dz \approx$ truncation independent $\Rightarrow$ Braginskii closures

General heat flux closure for sinusoidal temperature profiles: $T = T_0 - T_1 \sin(2\pi L/L_T)$



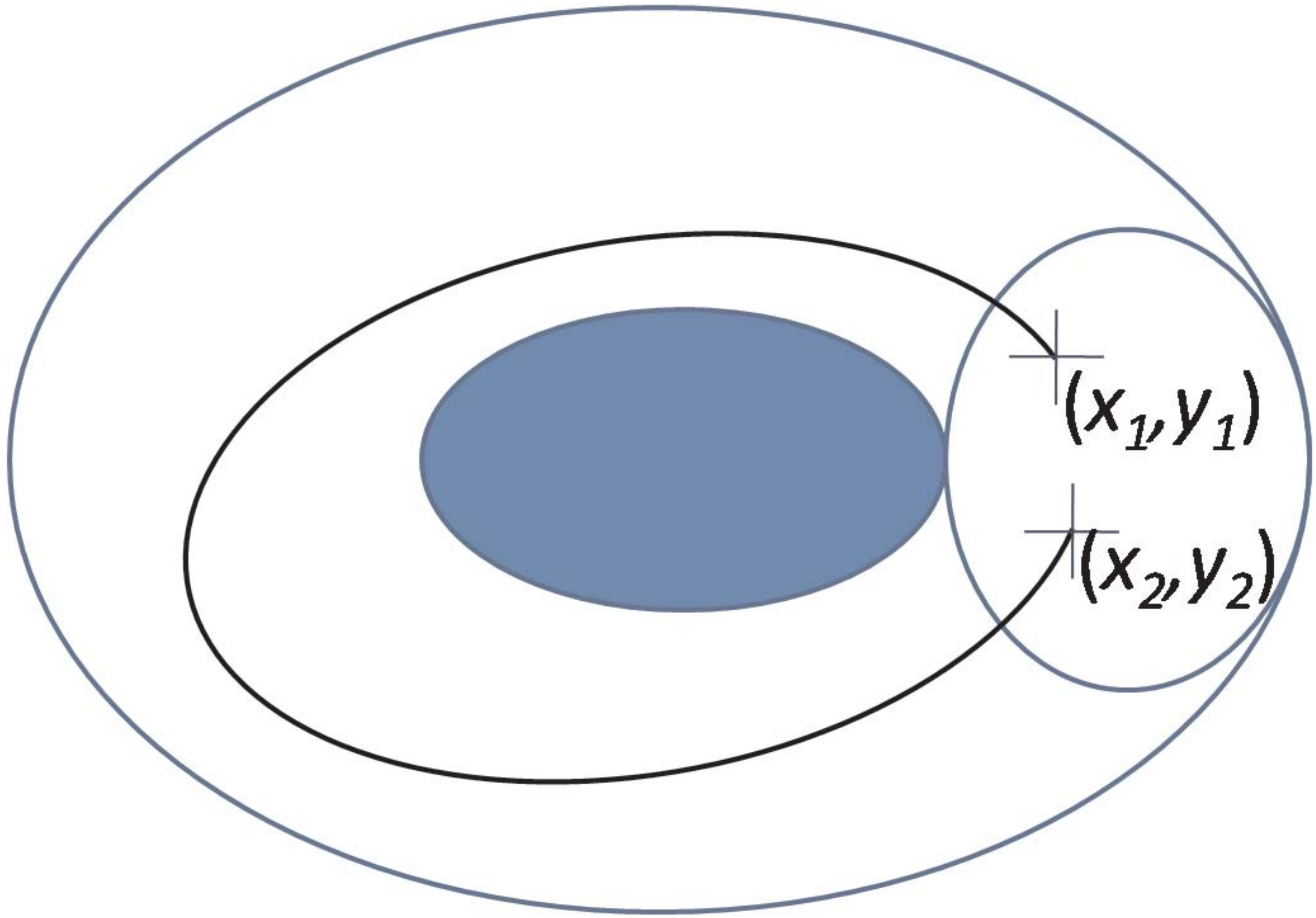
- Errors in Braginskii:
  $L_{\rm mfp}/L_T = 0.01 \Rightarrow 12\%$, $L_{\rm mfp}/L_T = 0.036 \Rightarrow 82\%$

General heat flux for a monotonically varying temperature: $T = T_0 - T_1 \tanh(L/L_T)$



- There is heat flow even when $dT/dL = 0$

NIMROD implementation



- Don't need to integrate from 0 to ∞ along a field line.
- At each toroidal section, calculate once the length of field lines $L(x_1, y_1)$ and closure integrations $I(x_2, y_2)$, then $I_{\mathrm{total}}(x_1, y_1) = I_1 + e^{-kL_2} I_2 + \cdots$

Summary and future Work

- The general parallel heat flux closure is derived.
  - Stationary general moment equations are analytically solved.
  - Full linearized collision operators are used.
  - Velocity integral is removed in the heat flux kernel.
  - Parallel viscous stress can also be obtained.
  - Closures with the electric field and flow gradient drives are on the way.
- NIMROD implementation and ...
  - Implement general closure equations in NIMROD.
  - Temperature simulation (NIMROD) in SSPX.
  - Numerically solve the time-dependent solutions with a number of moments estimated from the convergence property.