Two-fluid simulations of merging-compression start-up on the Mega-Amp Spherical Tokamak

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Mega Ampere Spherical Tokamak (MAST)

Normal Operating Parameters

- Major radius: \( R = 0.85 \) m
- Minor radius: \( a = 0.65 \) m
- \( R/a = 1.3 \)
- Toroidal field (at \( R \)): \( B_T = 0.5 \) T
- Current: \( \leq 1.6 \) MA
- Temperature \( \sim 0.1 - 3 \) keV
- Density: \( 10^{18} - 10^{20} \) m\(^{-3}\)
- Ion Species: Deuterium

- P3 coils: used for \textit{merging-compression start-up}
- P4, P5: vertical field, P6: vertical position
- Direct Induction (DI) start-up is expensive.
- Merging-Compression is an attractive alternative and is routinely used in MAST.
- Breakdown and induction around P3 coils.
- Merging via reconnection at mid-plane to form single ST.
- Compression via ramp-up of vertical field.
- Final state $I_{\text{plasma}} \propto I_{P3}$
- Up to 0.5 MA plasma current obtained.
- Up to $T_e = 1$ keV achieved in on ms timescale.
Merging-compression start-up

Time resolution: 0.1 ms. Total time: ~ 7ms.
Experimental Data: Thomson Scattering

Nd:YAG laser: 1D radial chord

- Upgraded 130 pt Nd:YAG TS
- Radial chord at midplane (Z = 1.5 cm)
- **0.1 ms** “burst-fire” mode
- Double peak feature in density profile
- $T_e$ increase from 10 eV to ~100 eV
- Central $T_e$ peak with $\Delta R$, $\Delta Z \sim 1$ cm.
- Oscillations ($\tau \sim 30 \mu s$) in CCMV20 signal during/after merging.

Data taken by T. Yamada (University of Tokyo) and the MAST team. See Ono et al. 2012.
Thomson Scattering: Electron heating

Ruby laser + P6 (vertical position) coils: 2D profile

- Each 2D profile built from several (identical) shots with different vertical shift.
- At later time (~5 ms after merging) electron temperature still increasing.
- Significant electron heating often occurs after merging in a “hollow” structure.
- The 1-10 ms time-scale is in agreement with electron-ion equilibration time

\[ \tau_{ie} \approx 0.2 \text{ ms} \times \left( \frac{T_e [\text{eV}]}{T_0} \right)^{3/2} \quad T_0 = 10 \text{ eV} \]
Filamentary structures in merging-compression

- Filaments visible in background-subtracted visible light images.
- Occur following spike in CCMV20 coil signal at central post.
Fluid model of merging-compression

Magnetic: \( B_p = 0.1 \text{T}, \quad B_T = 0.5 \text{T}, \quad I_T = 0.2 - 0.5 \text{ MA} \)

Thermal: \( T_e = 10 \text{ eV}, \quad n = 5 \times 10^{18} \text{ m}^{-3}, \quad \text{Deuterium} \)

\[ \begin{align*}
\beta_T &= 4 \times 10^{-5} < m_e/m_i \\
\beta_p &= 10^{-3}
\end{align*} \]

Initially \( S = 10^5 \) (\( \eta_\parallel = 3.5 \times 10^{-5} \Omega \text{ m} \)), Ion skin-depth: \( d_i = 15 \text{ cm} \), electron: \( d_e = 0.25 \text{ cm} \), Ion Larmor radius: \( \rho_i = \rho_{is} = 0.13 \text{ cm} \). c.f. Central peak in electron temperature ~ 1cm.

\[ \begin{align*}
\partial_t n + \nabla \cdot (n \nu_i) &= 0 \\
\partial_t (n \nu_i) + \nabla \cdot (n \nu_i \nu_i + (p_i + p_e) I + \pi_i) &= j \times B \\
E &= -\nu_i \times B + (d_i/n) (j \times B - \nabla p_e) + \eta j - \eta_H \nabla^2 j \\
\partial_t B &= -\nabla \times E
\end{align*} \]

- Hyper-resistivity is used to set dissipation scale for Whistler waves.
- Can (and does) set diffusion scale here by breaking frozen-in condition.

MAST merging-compression is a high \( S \), low \( \beta \) reconnection experiment.
Fluid model of merging-compression

**One temperature** formulation:

\[
(\gamma - 1)^{-1} \left[ \partial_t p + v_i \cdot \nabla p + \gamma p \nabla \cdot v_i \right] = \eta j^2 + \pi_i : \nabla v_i - \nabla \cdot q \tag{5}
\]

- \( \text{Re} = 10^3 \) (based on \( \mu_{ii} = 10^{-4} \, \text{Pa s} \).)
- Use \( \kappa_{ii}^\parallel \) and \( \kappa_{ii}^\perp \) based on initial \( n_0, T_0 \).

\[
q = -\kappa_{e}^\parallel \nabla_{\parallel} T - \kappa_{i}^\perp \nabla_{\perp} T
\]

\[
\pi_i = -\mu (\nabla v_i + \nabla v_i^T)
\]

**Two-temperature** formulation:

- Electron-ion equilibration time: \( \tau_{ie} > 0.2 \, \text{ms} \), longer than merge time.
- Temperature dependent resistivity (Spitzer parallel).

\[
(\gamma - 1)^{-1} \left[ \partial_t p_i + v_i \cdot \nabla p_i + \gamma p_i \nabla \cdot v_i \right] = -\pi_i : \nabla v_i - \nabla \cdot q_i \tag{5}
\]

\[
(\gamma - 1)^{-1} \left[ \partial_t p_e + v_e \cdot \nabla p_e + \gamma p_e \nabla \cdot v_e \right] = \eta j^2 - \nabla \cdot q_e \tag{6}
\]

- Use \( \kappa_{e}^\parallel, \kappa_{e}^\perp, \kappa_{ii}^\parallel, \kappa_{ii}^\perp \) based on initial \( n_0, T_0 \).
Modelling merging-compression start-up

- Solved in 2D (R, Z) with the **HiFi framework** (eg. Lukin and Linton 2011).

- **4th Order** spectral elements.
- Stretched grid: High resolution in current sheet.
- **Crank-Nicholson** ($\theta = 0.5$) time advance – avoid CFL condition on dispersive waves.

HiFi development and user support is provided through the **PSI-Center**.
Initial and Boundary Conditions

- Currently no q-profiles of pre-merged flux-ropes (plan for M9 campaign).
- Idealised initial conditions using $I_T = 0.2-0.5$ MA.

- Smooth, localised current profiles:
  \[
  j_T(r) = \begin{cases} 
  j_0 \left(1 - \frac{r}{w}\right)^2 & \text{if } r \leq w \\
  0 & \text{if } r > w 
  \end{cases}
  \]
  where $r^2 = (R - R_0)^2 + (Z \pm a)^2$.

- Balanced against pinch-force by $B_T$ increase ($\beta_p \sim 10^{-3}$).

- Perfectly conducting walls.

- Toroidal:
  - Line-tied vertical field $B_v = 0.03$ T.
  - Radial dependence of toroidal field.
Resistive MHD simulation: Cartesian geometry

- Increase in $B_T$ slows approach in ideal phase.
- **Force-free current-sheet**, Sweet-Parker like.
  \[ \Delta \sim w, \delta \sim S^{-1/2} \]
- Pile-up of $B_R$ on sheet edge, and reconnection stalls.
- **Sloshing** of flux-ropes, c.f. coalescence instability.
  (Biskamp & Welter 1980)
- Total merge time: $140 \tau_0 = 40.6 \mu s$. 
Two-fluid: Cartesian geometry

- Current sheet and outflow (ion) jets tilt.
- Order 1 density variations in “quadrupole” structure.
- Electrons accelerated in low density regions.
- Similar to standard picture of collisionless guide-field reconnection (e.g. Kleva et al. 1995, Ricci et al. 2004).

Cartoon from Kleva et al. (1995).
However, large aspect ratio current sheet is **unstable to island formation** (for $\eta_H \leq 10^{-8}$).

In Cartesian geometry a central island stalls the reconnection.

Stronger $B_T$: weaker density asymmetry. Multiple, shorter wavelength islands.

Peak reconnection rate in 2fluid 10x MHD rate. Total merge time 7-12 x faster.

Initial rate has a **weak dependence on dissipation** (hyper-resistivity).

Peak reconnection rate insensitive to $\eta_H$. 

The University of Manchester
Toroidal geometry breaks symmetry – island ejected (possible filament?).

Two-fluid merge time same as in Cartesian case ($= 25 \tau_0 = 10 \mu s$ for $\eta_H = 10^{-8}$).

Final state current profile similar for resistive and two-fluid simulations.
MHD: Density peak on inner side, cavitation on outer edge.

Two-fluid: Additional density asymmetry, disappears after merging completion.
Double peaked density profile. Evolution similar to Nd:Yag profiles.

Investigate further with 2D $n_e$ profiles (shifting plasma with P6 coils).

Initial peak in Mirnov does not correspond to merge time.

Simulated Mirnov is too fast (by factor of 3): other PF coils may be important.
Two-temperature formulation

- Ion temperature profile “hollow”. Tilt in ion temperature profile.
- Electron temperature ~ 100 eV, but no central peak.
Future Work: MAST M9 Campaign

**Experiments by H. Tanabe (University of Tokyo) and the MAST team**

- Measure 2D Ion Temperature profiles.
- Use Motional Stark Effect with neutral beam to constrain EFIT++ reconstruction.
- Neutral Beam Current Drive on merged plasma to reach > 1MA current.

**Simulation:**

- Comparison of pre and post-merge q-profiles with new data.
- Formulation with parallel ion viscosity, compare with new ion heating data.
Conclusions

▶ Strong toroidal field slows merging in both ideal and resistive phases.

▶ Tilt of outflow jets, and ion temperature profiles in two-fluid case.

▶ Current sheet is large aspect and unstable to secondary island formation.

▶ Cylindrical geometry has little effect on reconnection process (eg. the reconnection rate) but can modify density profiles.

▶ Simulated 1D density profiles show same time evolution as in TS profiles.